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The importance of the effects of diffraction and focusing on current deposition of lower hybrid waves

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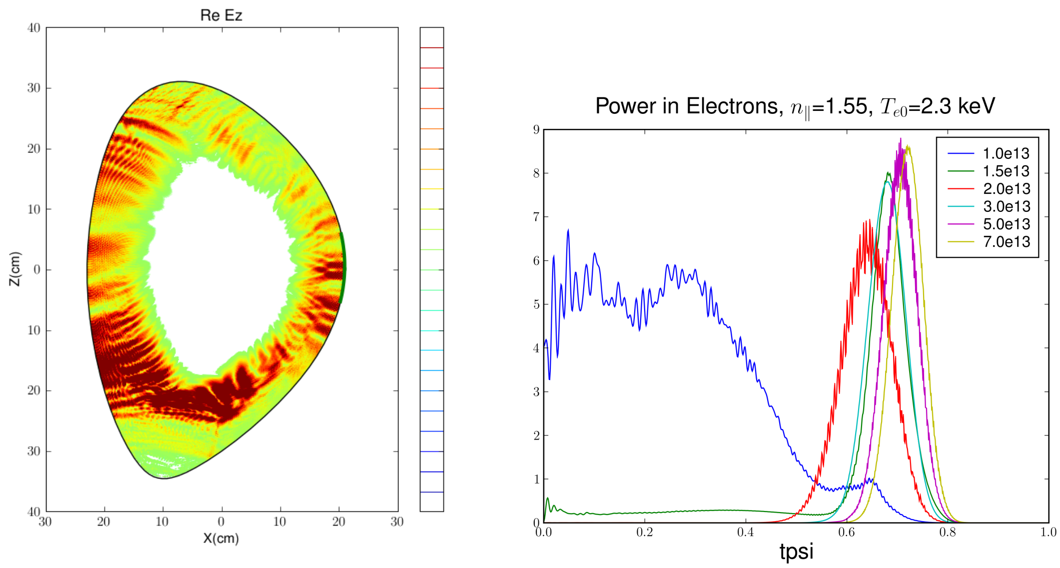
Introduction

Lower hybrid (LH) waves have the attractive property of damping strongly via electron Landau resonance on relatively fast tail electrons at 2.5 v_{te}, where $v_{te} = (2T_e/m_e)^{1/2}$ is the electron thermal speed. Consequently these waves are well-suited off-axis (r/a>0.60) current profile control in reactor grade plasmas. It is therefore important to develop a predictive capability in this area. Advanced LH simulation codes treat wave propagation in the geometrical optics limit using toroidal ray tracing, which is known to neglect important effects on the wave spectrum due to focusing and diffraction [1]. In order to accurately assess these effects we have developed a parallel version of the TORIC full-wave electromagnetic field solver valid in the LH range of frequencies with a non-Maxwellian electron dielectric [2, 3]. The diffraction observed in the full wave code is sufficiently strong to downshift the wave phase speed, causing the LH waves to damp at $r/a \approx 0.75$ in a 4 keV plasma. We note the full-wave treatment described in this paper should be more accurate than methods that employ paraxial beam tracing algorithms [4] or made approximations in size or geometry, or expanded in small parameters [5, 6, 7].

At the lower hybrid (LH) frequency, defined by $\Omega_{\rm ci} \ll \omega \ll \Omega_{\rm ce}$, where $\Omega_{\rm i,e} \equiv qB/m_{i,e}c$ are the electron and ion cyclotron gyration frequencies in a magnetic field of strength B, plasma waves are nearly electrostatic and have very short wavelengths relative to equilibrium scale lengths. The waves are thus good candidates for a WKB approach such as ray tracing which has been the solution method of choice. However there are several known deficiencies with this approach. Lower hybrid waves are weakly damped and undergo multiple reflections from the low density cutoff at the edge of the plasma. The rays also propagate along characteristics of the electrostatic wave equation known as resonance cones that tend to become narrow and even singular at turning points forming caustics when they encircle the axis. Extended ray tracing techniques such as the Maslov method popular in seismology [8] and the wave-kinetic method [9], are valid at the caustic surfaces; but because the LH cutoffs in tokamak plasmas occur in the plasma edge where the gradients are very large, they violate the WKB approximation where the plasma is changing on the same scale as the wavelength [10].

A full wave approach that solves the Maxwell-Vlasov system directly will not be subject to these restrictions, and it will retain other physical processes that may be important to the propagation and damping of the waves. In this paper we investigate the importance of diffraction in lower hybrid propagation using an adapted version of the TORIC code[11]. The TORIC-LH[3] version has been modified to solve for the fast and slow branches of the lower hybrid wave at frequencies above the lower hybrid frequency. We believe these results demonstrate the first full wave calculations of LH waves in toroidal geometry of a full scale tokamak without the above approximations.



(a) Full wave calculation of lower hybrid waves in Alcator C-Mod at electron density of $7 \times 10^{13}/cc$

(b) Dependence on density of power deposition in full wave calculation of lower hybrid waves in Alcator C-Mod

Figure 1: Core penetration of LH waves is a sensitive function of density. All simulations were done with the reconstructed magnetic equilibrium from an Alcator C-Mod discharge and used a non-Maxwellian electron distribution with a quasilinear tail.

The full wave approach

The full wave equation is given by

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \left\{ \mathbf{E} + \frac{4\pi i}{\omega} \left(\mathbf{J}^{\mathrm{P}} + \mathbf{J}^{\mathrm{A}} \right) \right\} \tag{1a}$$

$$\mathbf{E}(\mathbf{x}) = \sum_m \mathbf{E}_m(r) \exp\left(im\theta + in\phi \right) \tag{1b}$$

$$k_{\parallel} = \left(m\mathbf{B} \cdot \nabla\theta + n_{\phi} \mathbf{B} \cdot \nabla\phi \right) / B \tag{1c}$$

$$\mathbf{J}^{\mathrm{P}}_m(r) = \sum_m \overleftrightarrow{\sigma}_c \left(k^m_{\parallel}, r \right) \cdot \mathbf{E}_m(r) \tag{1d}$$

where Eq. 1a is solved by a variational technique[12] that results in a block tri-diagonal stiffness matrix. The electric field is expressed in the basis given in Eq. 1b has the advantage of providing an algebraic expression for the parallel wavenumber in Eq. 1c which is useful in evaluating the the plasma dielectric, $\overleftrightarrow{\sigma}_c$ needed to determine the plasma current response to the radio frequency waves as given in Eq. 1d. The radial dependence is represented by finite elements using cubic Hermite polynomials as basis functions. The plasma dielectric is a function of the equilibrium electron distribution. TORICLH uses a non-Maxwellian distribution that can be specified or calculated by a Fokker-Planck code[13].

Full wave effects

Calculation of propagation with the full wave approach show striking similarities and differences from the ray tracing approach. Striations form in the field patterns such as in Figure 1(a) that we identify as resonant cone structures[14] and are also seen in the trajectories of rays from WKB codes. What is also evident is that the wave fields do not reach very far into the plasma. The waves are accessible by the LH accessibility criteria but are simply absorbed far from the core plasma. Analysis of the wavenumber spectrum of the solution confirms this is the case. This is different from the behavior typically seen in ray tracing and Figure 2 shows the contrast in radial power deposition in the two approaches.

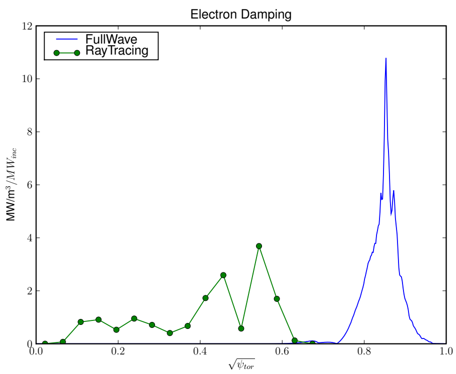


Figure 2: Comparison in location of power deposition between ray tracing and full wave calculations for the Alcator C-Mod case at a density of $n_e(0) = 7 \times 10^{13}$/cc, $n_{\|} = 1.55, N_m = 1023, N_r = 480, T_e(0) = 2.3keV, B(0) = 5.4T$

A likely mechanism for this enhanced spectral broadening and acceleration of the filling of the spectral gap is the presence of diffraction in the full wave solver. The resonance cones that narrow as the waves penetrate the plasma can act as sources of diffraction [15] as can caustic surfaces that are seen in full wave and ray tracing. There are plasma parameters for which the full wave calculations still show core absorption. In particular, the penetration to the core is a sensitive function of density. Figure 1(b) shows that the power remains excluded from the core until the density drops below $2 \times 10^{13}/cc$ when it shifts to core absorption. The resonance cone angle does depend on density and there may be a relation between this angle and the onset of spectral broadening.

Conclusions

We have shown calculations for the first time of LH waves in toroidal geometry in a laboratory scale device. Realistic general geometry and non-Maxwellian electrons were used. These simulations predict quite different absorption locations for the waves than traditional ray tracing approaches. The differences are large enough to be validated experimentally. The next step in this work will be to complete the iteration with a Fokker-Planck code for self-consistency and generate synthetic hard X-ray spectra for direct comparison to experiment.

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